



The Evolution of Triple Black Hole Systems

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INTRODUCTION

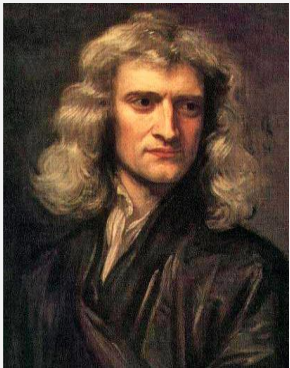


Fig. 1. Sir Isaac Newton.¹

The dynamics of three bodies dates back to Newton's *principia*. The problem is easily stated: How do three bodies move under the gravitational influence of each other? The answer, however, is far more complicated. This problem has been studied extensively and here we apply it to black holes.

What are the dynamics of a triple system of black holes? This question becomes relevant as we step into the frontier of detecting and analysing gravitational waves from detectors like

LIGO and VIRGO. The sources of these gravitational waves are mergers of compact objects and in most cases, black holes.



Fig. 2. Simulation of Black Holes merging.²

The study of triple black hole systems is moving from purely theoretical to observational as triple mergers of galaxies have been found. NGC6240, in Figure 3, shows a galactic merger which is thought to contain three supermassive black holes.



Fig. 3. NGC6240.³

Newtonian systems (small mass) follow similar trajectories, but relativistic systems (large mass) do not. The research presented here studies the influence of mass and distance on the evolution of such triple systems of black holes with the use of numerical simulations.

The three main parameters we focus on are: 1. The number of mergers that occur; 2. the number of binary encounters/interaction among the three bodies; and 3. the lifetime of the systems (time until the first merge, or in cases where mergers do not occur, the time until binary pair formation and mass ejection).



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METHODOLOGY

Following ideas from Burrau⁴ (1913) and Valtonen et al.⁵ (1995), we re-analyse Burrau’s problem of three bodies where the mass unit, m , and distance unit, d , reflect that of a 3,4,5 Pythagorean triangle (See Fig. 4.).

How does such a system move and evolve gravitationally?

This is answered with the use of Prof. Seppo Mikkola’s FORTRAN code, ARCcode⁶. Initial conditions are placed into the code and the position co-ordinates (w.r.t. to the center of mass of the system) are calculated (using 2.5th order Post Newtonian equations).

We can then use these co-ordinates to plot the orbital paths of the system as in Fig. 5.

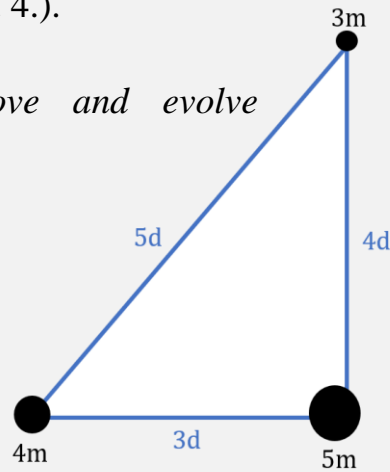


Fig. 4. Burrau’s problem of three bodies.

We study how Burrau’s three body problem evolves with time i.e. the initial placement of bodies (black holes) are as Fig. 4. We then extend this study to fifteen more Pythagorean triangles.

For each triangle we run 13 simulations with mass unit, m , ranging from 10^0 to 10^{12} solar masses. The effect of distance unit, d , was also investigated where d was varied from 0.01 to 10000 parsecs.

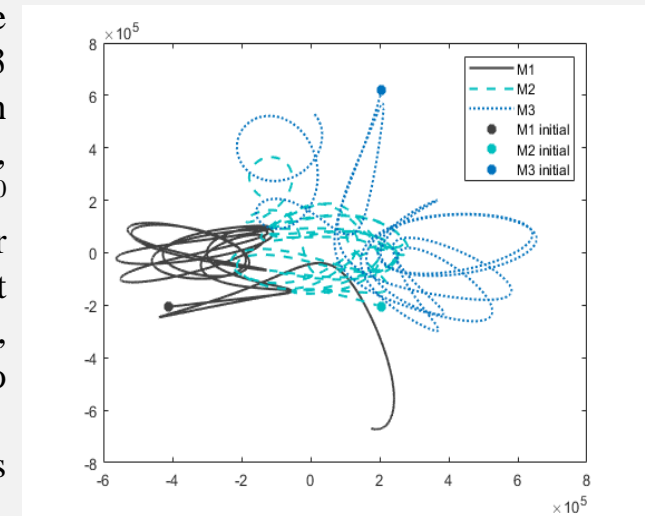


Fig. 5. Trajectory of Burrau’s three body problem with mass unit of 10^5 solar masses for time 0-2.2 million years.



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RESULTS & ANALYSIS

Chaos is inherent in the three-body problem. Changing some small parameter in a three black hole system for example can be the difference between two black holes merging quickly or not at all. To study triple systems of black holes then, we apply statistics to many simulations.

For Figures 5. and 6., we combine data from all triangular configurations. Mass is in solar masses i.e. the mass of our Sun.

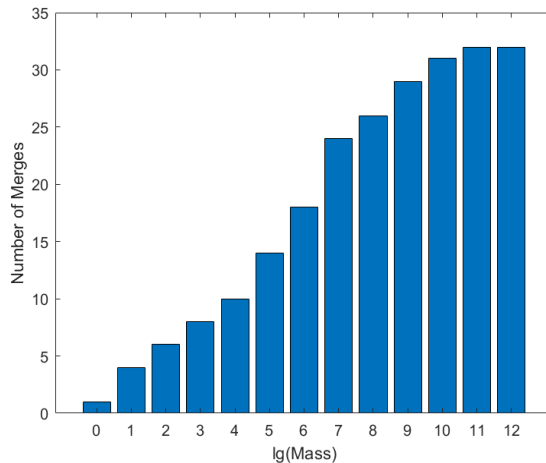


Fig. 5. Number of Merges vs. lg(Mass Unit).

From Fig. 5., as mass is increased the number of merges of black holes increase.

This is in accord with theory: the larger the mass, the greater the effect of gravity and subsequently, the greater the probability of black holes coming closer and closer and eventually merging.

The opposite occurs with binary encounters. As mass is increased and merging of all bodies dominate the systems, there is less interaction and movement among the bodies.

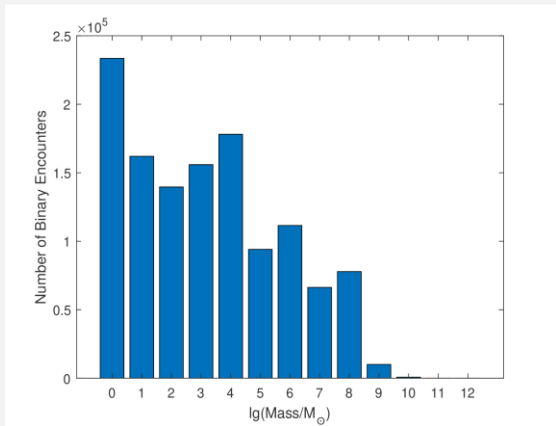


Fig. 6. Number of Binary Encounters vs. lg(Mass Unit).

We find a strong positive correlation between the number of merges and mass (0.9868), and a strong negative correlation between binary encounters and mass (-0.9494).



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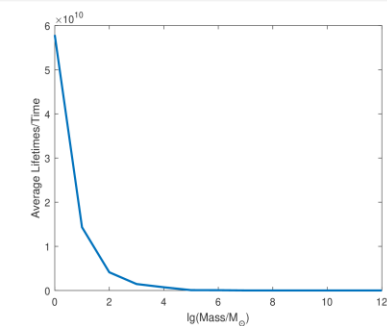


Fig.8. The Average Lifetimes of the Systems vs. $\lg(\text{Mass Unit})$.

The lifetimes of the systems decay exponentially (after curve fitting data, we obtain the equation to describe the lifetime of the systems under study $5.782 \times 10^{10} e^{-1.372x}$) as in Fig.8.

From Fig. 9., as the distance unit is increased for the smaller mass systems, the number of merges is reduced in the (9,40,41) startup configuration. This is until a mass unit of 10^6 solar masses, where even at a distance unit of 10000 parsecs, merging of all three black holes dominate the outcome.

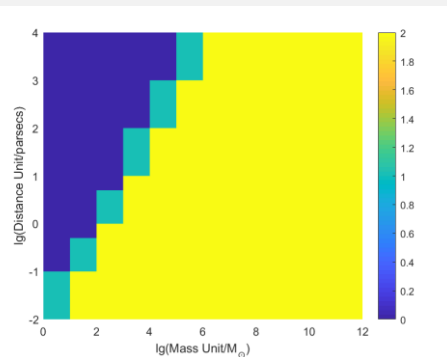


Fig. 9. The Number of Merges as Mass and Distance are Varied for (9,40,41).

Conclusion

The effect of mass on the evolution of triple systems of black holes shows that the larger the mass unit of the systems the greater the number of merges. The number of binary encounters as well as the lifetimes of the systems decrease as the mass unit is increased. The effect of distance unit, in the (9,40,41) triangle, on the number of merges is significant for smaller systems – as distance unit increases, the number of merges decrease – however, as mass increases past 10^6 solar masses, the effect of mass dominates, and merging takes precedence.

References

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